A Theory of 'The Loop': Policy-making and Information Aggregation through Networks*

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Abstract

We describe a model of strategic, decentralized and asynchronous communication in policy-making networks. Two central focuses of the model are the actors' awareness of who other actors will talk to in the future and the sequential ordering of actors' communications. We derive conditions for truthful "cheap-talk" communication within sequential communication networks and show that (1) the ordering of individuals within the network can matter above and beyond individuals' policy preferences and degree of decision-making authority, (2) sequential communication throughout can engender credible communication in situations in which private, dyadic communication will not, and (3) sequential communication can sometimes undermine credible communication, so that exclusion of one or more "extreme" (or extremely powerful) individuals from the communication network can be (Pareto) optimal. Finally, the analysis and results suggest that it is theoretically impossible to cleanly hive off homophily from the study of strategic information transmission in networks.

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1 Introduction: Networks and Communication

The motivation of this article is a simple syllogism: communication is central to politics, and networks are the foundation of communication. It is well-known that communication situations can give rise to counterintuitive strategic incentives. Arguably the most ubiquitous and bedeviling of these is the persistent ability of even minuscule amounts of preference diversity to undermine credible communication, or "truthful signaling," of information between two or more agents.[1] In spite of the breadth of the menagerie of situations considered in this body of work, the conclusions of all of these models share a clear, fundamental, and initiative foundation: credible communication rests largely on preference similarity between the message sender and message receiver. The analysis and conclusions presented in this article do not controvert that regularity: truthfulness is more credible when the sender and receiver desire more similar policy outcomes.

A limitation of many (but by no means all) of the existing theories of signaling is that they largely focus on two-player signaling situations. Clearly this is the canonical building block or starting point for more general theories of signaling, but the parsimony of these situations comes at the expense of obviating more detailed concerns about sequential communication. In this article, we consider one of the fundamental characteristics of such settings: the order of communication between individuals. While it is clearly possible for one agent to signal to several other agents without using an intermediary (for example, either by speaking with a single message to a group of other individuals or sending specialized/differentiated messages to multiple individuals or (sub)groups), sequential communication is both theoretically relevant and a practical reality. Of particular interest in both formal and informal environments are what one might call "one-at-a-time" messaging processes where one individual sends a message to another individual who then chooses how to convey what he or she heard to another individual, who may then convey what he or she heard from the second individual to a third agent, and so forth. For example, a formal institution that mimics this structure is the "client-attorney-judge" triad at the heart of criminal justice in the United States: the client possesses information relevant to the case at hand, which or she can choose to attempt to convey to his or her attorney, who then may choose how best to convey this information to the judge presiding over the case.[2] Informal situations possessing this structure are too numerous to mention: as illustrated by the children's game of "telephone," simple observation immediately confirms that

[1]The relevant literature on this topic is vast and broad. Particularly on point include the variety of models that consider "cheap talk" communication. These are situations in which the "credibility of the message" must be induced through its informational content as opposed to the sender's direct preferences over the choice of messages. The difficulty of achieving credibility in such settings has been established in wide variety of institutional and informational settings (*e.g.*,Crawford and Sobel (1982), Gilligan and Krehbiel (1987), Gilligan and Krehbiel (1989), Austen-Smith and Banks (1996), Farrell and Rabin (1996), Feddersen and Pesendorfer (1998), Aumann and Hart (2003), Levy and Razin (2007), Ambrus, Azevedo and Kamada (2013)).

[2]Of course, this microstructure is common to a wide array of political situations, including all of those in which an individual has "counsel" assisting him or her in representing his or her interests. Beyond criminal and civil court cases, examples include participation in administrative proceedings (*e.g.*, formal hearings and "informal rulemaking" processes) and legislative hearings.

much of social communication occurs sequentially.

A defining characteristic of any individual within such a setting is the individual's *position* in the communication process. That is, some individuals receive and send messages (*i.e.*, are "spoken" to and speak) earlier than others. For simplicity, we consider situations in which this ordering is fixed and known during the communication process.[3] That is, each individual has a unique position in the network and, furthermore, knows not only his or her position in the network, but also the positions of every other individual. In this sense, the analysis and results are arguably more proximate to formal communication networks/hierarchies/"chains of command."

In addition, and partly in line with that recognition, we focus attention on sequential communication when the individuals have (possibly varying degrees of) unilateral decision-making authority. Specifically, each individual, regardless of his or her position in the communication network

The Empirics of Communication Networks. In this article, we focus on individuals' incentives to be truthful within a communication network. This lever is partly analytical in nature: by considering a "thin" informational environment (the information that can be transmitted is binary, as in the answer to a "yes/no" question), there are essentially only two classes of behavior: either tell the truth or not. This is because not telling the truth is equivalent to "always saying the same thing" in this setting. But this starkness is in appearance only: if one moved to a richer informational environment, the incentives identified here would remain qualitatively the same, but the analytics would quickly become much more cumbersome without much additional substantive insight.

In addition, the focus on truthfulness can be motivated from an empirical standpoint. In a nutshell, less truthful communication is—in equilibrium—less valuable to *both* sender and receiver. This is because, without one or more individuals being strategically ignorant, less truthful messages are recognized as such and accordingly lead to less information transmission. In other words, if an adviser does not have an incentive to be truthful with the advisee, the advisee will—again, in equilibrium—place commensurately less weight on the adviser's advice when the advisee ultimately makes his or her policy choices.

Because of this, it seems reasonable to suppose that real-world communication networks, which are at least partially endogenously determined, will tend to be created and sustained between individuals whom have an incentive—given the network structure—to be truthful with each other. From a "network perspective" this is important for three reasons. First, the analysis and results presented below illustrate that the incentive for truthful communication depends on the network structure. In the settings considering this article, this amounts to another take on the ages-old adage that "order matters." Specifically, while the match between the preferences of the sender and receiver matters in engendering the credibility of truthful communication, introducing the possibility of subsequent

[3] Obviously, relaxing this assumption is desirable for many reasons. However, the usual constraints of space and time force us to leave this extension for future work.

further communication of the message implies that this credibility also depends on the positions of not only the sender and the receiver in the network, but also the positions of the other individuals. Succinctly, each individual needs to worry not only about how the individual he or she is speaking to will use the information, but also both how subsequent individuals will use the information *and* whether messages sent subsequently will be seen as credible. Thus, credible truthfulness at any stage of the communication process is dependent not only only the match between the preferences of the sender and those of the potentially many other individuals beyond the immediate receiver of the sender's message, but also on the matches between the preferences of the various subsequent individuals.[4]

Second, many communication networks are either directly observed or inferred by various indirect methods, such as surveys. To the degree that actively participating in a communication network is voluntary and has some (opportunity) costs, one should suspect that observed networks will tend to be those that make truthful communication credible. This has potentially important implications for attempts to infer the welfare/normative effects of network structure on information diffusion.[5] There are two related subpoints here. The first is a classic selection effect: if empirically observed communication networks tend to be those that engender credible truthfulness, then making inferences about the effect of network structure *per se* on (say) information diffusion can lead to dramatically incorrect conclusions. (Of course, if one qualifies the conclusions from such an analysis as something like "the estimated effect of network structure on information diffusion among networks that engender credible truthfulness," then the selection effect is obviated and the inserted qualification clearly indicates the relevance of the analysis reported in this article.) The second subpoint is more specifically focused on empirical studies of networks and deals with *homophily*: the tendency of individuals with similar characteristics to associate with one another. In particular, the results reported here clearly indicate that *it is impossible to cleanly hive off homophily from the study of strategic information transmission in networks*. Put succinctly, to the degree that homophily is proximate to preference similarity and truthful communication is more valuable than muddled or obfuscated communication, homophily is inevitably positively associated with observing a communication network. More subtly, this conclusion is reached in spite of the fact that we do not assume there is any direct preference for communicating with similar individuals: in fact, in the model analyzed in this article, every individual strictly prefers (credible) truthful communication with *any* other individual and, more importantly, each individual is strictly indifferent between there other potential

[4]Furthermore, from a slightly different perspective, the ability of any given individual to convey information (setting credibility aside for the moment) is dependent on the analogous preference alignment calculations for those who sent messages prior and up to the sender receiving his or her message: in the settings analyzed here, information is "lost" for all subsequent senders and receivers once one individual lies/obfuscates.

[5]Examples of recent analyses of the effect of informal network structure include: the impact on knowledge transfer (Reagans and McEvily (2003)), the diffusion of political information on twitter (Romero, Meeder and Kleinberg (2011)), information propagation through Flickr (Cha, Mislove and Gummadi (2009)), the sharing of information in online social networks (Bakshy et al. (2012)), and the structural determinants of social capital provision (Burt (2000)).

communication partners. Thus, the model generates "observed homophily" without individuals having pro-homophily/congruence preferences. Rather, the homophily effect emerges to the degree that homophily is correlated with credibility of communication.

Third and finally, the analysis and results reported here indicate that communication networks not only diffuse information—they also can simultaneously engender credible communication and propagate credibility problems. In particular, a communication network can engender credible communication—for example, in situations in which an individual i would not have an incentive to communicate truthfully to another individual j in a two-player setting but i does have an incentive to communicate truthfully to j when j will subsequently have the opportunity to pass i's information along to a third individual, k—or it might undermine credibility, such as in cases in which an individual i could credibly and truthfully communicate with j so long as j could not "pass i's message along" to some other third individual, k, but i could not be credibly truthful with j so long as j would subsequently have an opportunity to communicate with k. Somewhat ironic about this undermining dynamic is that it is a direct result of j's ability to credibly communicate truthfully with k. In other words, credible communication at later positions in a network structure can pathologically overturn the credibility of messages earlier in that network. From a "networks matter" standpoint, this point further highlights the importance of understanding both the reality, and individual actors' beliefs/knowledge, of the structure of the communication network within which they are embedded.

Now, prior to moving to the model, it is useful to discuss a few recent theoretical models of information transmission that are related in various ways to the model developed and presented in this article.

1.1 Related Literature

The framework utilized in this article assumes that individuals have a noisy but informative signal about an underlying state of nature. Thus, it is related to the models presented in Austen-Smith (1993), Wolinsky (2002), and Battaglini (2004). The theory presented in this article differs from those in that the information to be aggregated and potentially messaged is held by the same individuals who will make decisions. This distinction is particularly relevant when considering institutional design and welfare issues, as it renders impossible attempts to mitigate informational problems by simply picking a better advisor (typically one whose preferences are more consonant with those of the decision-maker). It is also a more realistic construction of the practical design problem faced when considering, for example, how to organize a self-regulating body within an industry.

The contributions of Hagenbach and Koessler (2010) and Galeotti, Ghiglino and Squintani (2013) are most closely related to the theory presented in this article. Each of these articles considers information transmission through networks with decentralized policy-making embedded in similar preference and information environments utilized in this article. Hagenbach and Koessler (2010) considers a different informational setting that is more general in that it allows for agents' signals to be of heterogeneous qualities (*i.e.*, some agents' signals are more informative about the underlying state of nature than others), but also imposes a constant marginal impact of truthful signals.[6] While their decision-making is very similar to that examined here, they focus on a common coordination incentive between agents that is qualitatively different from the incentives considered in this article. More importantly, Hagenbach and Koessler (2010) consider voluntary unilateral transmission of messages in the sense that each agent can decide to whom he or she wishes to send a message: thus, the communication network is endogenously generated in their framework.[7] The combination of these two features (coordination and endogenous network structures) implies that there is no general Pareto dominance relation between equilibria involving communication by different numbers of agents (Hagenbach and Koessler (2010), p. 1078).

Galeotti, Ghiglino and Squintani (2013) examine information aggregation through exogenously-specified network structures. By considering directed networks, their framework allows for the particularly interesting possibility of "one-way" communication, in which one agent i is able to send a message to agent j, but agent j is prohibited from sending a message to agent i. Galeotti, Ghiglino and Squintani (2013) presume (as do Hagenbach and Koessler (2010)) that each agent has equal decision-making authority in the sense that each agent's policy decision has the same impact on every other agents' payoff. By relaxing this assumption, the framework considered in this article allows for what we refer to as "purely advisory" agents, whose only impact on social welfare is through their private information as carried through the (equilibrium) impact of their messages on the policy choices of other agents with positive decision-making authority.

The model utilized in this article is also closely related to that of Dewan and Squintani (2012), who use the same informational environment to consider the creation and allocation of power within political factions. Dewan and Squintani (2012) focus on the question of how decision-making authority might be transferred in equilibrium between agents prior to information aggregation in pursuit of more-informed (equilibrium) policy-making. Gailmard and Patty (2019) consider the potential impact of both endogenous power-sharing/delegation and transparency in a model of sequential decision-making. Patty and Penn (2014) similarly consider sequential decision making and information aggregation and focus on the incentive and welfare impacts of different (small) network structures.

[6]Formally, the state of nature is equal to the sum of the agents' signals.

[7]Formally, a communication link from agent i to agent j if agent i plays a pure separating (*i.e.*, perfectly informative) strategy in terms of the messages agent i's sends to agent j.

2 The Model

Let N denote a set of n individuals, $X = \mathbf{R}$ denote a policy space, and $\Theta = [0,1]$ denote a state space. Each individual $i \in N$ is initially (or formally) endowed with policy making authority $\alpha_i \geq 0$, which measures the degree of unilateral decision-making autonomy possessed by agent $i \in N$. The state of nature, $\theta \in \Theta$, is determined according to a distribution characterized by cumulative distribution function $F : [0,1] \to [0,1]$. Upon realization of θ according to F, exactly one individual $\sigma \in N$ receives a conditionally independent (and private) signal $s_\sigma \in \{0,1\}$. The player that observes the signal, σ, is referred to as the *source*. The source's identity is determined as follows: with probability p_i, agent i observes the signal (and nobody else does) and $p \equiv \{p_i\}_{i \in N}$ satisfies the following: $\sum_{i \in N} p_i = 1$ and $p_i \geq 0$ for each $i \in N$.[8] After realization of the source, σ, the binary signal observed by him or her is realized according to the following probability mass function:

$$\Pr[s_i = x | \theta] = \begin{cases} 1 - \theta & \text{if } x = 0, \\ \theta & \text{if } x = 1. \end{cases}$$

Letting $g_i(\cdot|s_i)$ denote the probability density function of i's posterior probability distribution function of θ, given $s_i \in \{0,1\}$, this belief is given by

$$g_i(t|s_i) = \begin{cases} \frac{F(t)(1-t)}{1-E_F[\theta]} & \text{if } s_i = 0, \\ \frac{F(t)t}{E_F[\theta]} & \text{if } s_i = 1. \end{cases} \tag{1}$$

For simplicity, we assume throughout that F is the cumulative distribution function for the Uniform$[0,1]$ distribution. This assumption implies the (**??**) reduces to the following:

$$g_i(t|s_i) = \begin{cases} 2(1-t) & \text{if } s_i = 0, \\ 2(t) & \text{if } s_i = 1. \end{cases}$$

Messaging Through A Network. Throughout this article, we consider a tertiary messaging technology, where each message any individual sends must be "0," "1," or "φ," which we will interpret as "no message."[9] As in Galeotti, Ghiglino and Squintani (2013) and Patty and Penn (2014), we consider *messaging networks*. In line with Patty and Penn (2014) and in contrast to Galeotti, Ghiglino and Squintani (2013), we consider a class of messaging networks in which the messages are conveyed sequentially. The communication network (or *graph*) is denoted by $G \subset N^2$, where $(i,j) \in G$ implies that agent i can send a message to j. Furthermore, for reasons of tractability and notational

[8] We do not utilize this parameter in the analysis reported in this article. Rather it is included to facilitate description of an interesting possible extension for the model, which we discuss in Section 4.3.

[9] This labeling is arbitrary, but we will simply note and leave to the side the question of equilibrium refinements with respect to off-the-path beliefs. See Patty and Penn (2014) for more on the issue of such beliefs in a related setting.

simplicity, we consider a special form of communication network, a *non-crossing loop*, satisfying the following. For each agent $i \in N$, there is exactly one agent $j \neq i$ such that $(i,j) \in G$ and, for each pair of agents i,k, there exists a path between i and k in G. In other words, there exists some finite m such that there exists a sequence of edges in G, $\{(i,j_1),(j_1,j_2),\ldots,(i_{m-1},k)\}$.

The communication network, G, is realized prior to (*i.e.*, independent of) the realization of the source's identity, $\sigma \in N$.

Payoffs. After the communication round, each player $i \in N$ independently and simultaneously chooses $y_i \in \mathbf{R}$, with $y = (y_1,\ldots,y_n)$, and has a payoff function of the following form:

$$u_i(y,\theta;\beta) = -\sum_{j=1}^{n} \alpha_j (y_j - \theta - \beta_i)^2,$$

where $\beta_i \in \mathbf{R}$ denotes the *preference bias* of agent i and $\beta \equiv \{\beta_i\}_{i \in N}$ denotes the profiles of all preference biases. We assume throughout that these biases are common knowledge to all of the players. Note that the autonomy of each player j factors into the payoffs of every player (including j) by determining the importance of j's decision. Thus, setting $\alpha_j = 0$ is equivalent to eliminating j's decision-making authority.

Policy-making. Following the messaging stage, each individual is presumed to make unilateral decisions that are private in the sense of not being observed by any other agent until after all agents' policy decisions have been made. Thus, policymaking in equilibrium will always be "truthful," because one's policy choice cannot affect the policy choices of any other agents.[10]

A player's posterior beliefs after m trials and k successes (*i.e.*, k occurrences of $s = 1$ and $m - k$ occurrences of $s = 0$) are characterized by a Beta$(k+1, m-k+1)$ distribution, so that

$$\begin{aligned} E(\theta|k,m) &= \frac{k+1}{m+2}, \text{ and} \\ V(\theta|k,m) &= \frac{(k+1)(m-k+1)}{(m+2)^2(m+3)}. \end{aligned}$$

Accordingly, the optimal policy choice for a policymaker, given (truthful) revelation of k successes and $m - k$ failures, is

$$y_i^*(k,m) = \frac{k+1}{m+2} + \beta_i. \tag{2}$$

[10]This distinguishes this article's analysis from those in both Patty and Penn (2014) and Gailmard and Patty (2019).

2.1 Strategies and Equilibrium

We focus on pure strategy perfect Bayesian equilibria (referred to more simply as an equilibrium) in this article.[11] Furthermore, we focus on a refinement of these equilibria in which, if any agent i has an incentive to be truthful to agent j given that agent j believes that agent is being truthful, agent j believes that agent i is truthful. This refinement clarifies the role of excluding one or more players from the network in order to support truthful communication by those who are retained in the network. We refer to this refinement as the *listening perfection refinement*, and mention when it plays a substantive role in our analysis.

For each individual $i \in N$, i's strategy consists of a messaging strategy, $\mu_i : \{0, 1, \varphi\} \to \{0, 1, \varphi\}$, and a policy-making strategy, y_i.[12] Sequential rationality in equilibrium pins down y_i^* as described in Equation (2). Accordingly, we characterize equilibria entirely by the vector of players' messaging strategies. Each player i has three possible information sets: $\mathcal{I}_i \equiv \{0, 1, \varphi\}$, denoting the value of the signal or message observed by i, and denote an arbitrary element of $\mathcal{I}_i$ by η_i.[13]

For any given non-crossing loop network G with n players and a given source $\sigma \in N$, the set of possible pure strategy equilibria can be partitioned into n equivalence classes, depending on many truthful messages are sent. Among the equilibria satisfying the listening perfection refinement, it is without loss of generality to presume that every agent i who observes the "no message" message φ will similarly transmit $m_i = \varphi$ to the next agent in the network. This is for two reasons. First, the labels of the messages are *per se* unimportant in this cheap-talk setting, so that it is appropriate to focus on equilibria in which truthful behavior involves choosing $m_i = \eta_i$. Second, if agent i is sending a message to agent j and truthfulness by agent i is not incentive compatible given that agent j believes agent i is being truthful, it is without loss of generality to presume that agent j's beliefs are invariant (*i.e.*, unchanged by) agent i's choice of message and, accordingly, agent i always sends $m_i = \varphi$, regardless of η_i. In other words, in the equilibria we focus on here, when cheap-talk between i and j is not incentive compatible, both agents recognize this fact and simply "don't talk," with the message sent between them always being the "no message" message. Thus, if player i observes φ, it is common knowledge that there is no information contained in that message, and every agent subsequently has a weak incentive to truthfully reveal that he or she in fact "knows nothing."[14]

[11] This approach is also used in Hagenbach and Koessler (2010), Galeotti, Ghiglino and Squintani (2013), Dewan and Squintani (2012), Gailmard and Patty (2019), and Patty and Penn (2014). Mixed strategy equilibria can exist in these settings, but characterization of such equilibria is very difficult due to the combinatorics of the underlying problem.

[12] Note that $\mu_i(\varphi)$ is relevant only if i is not the source, σ.

[13] For simplicity, we treat the observation of (say) a signal $s_i = 0$ and a message equal to 0 as equivalent and similarly for $s_i = 1$. This is technically an abuse of notation, as each player is assumed to know whether he or she is the source, and therefore know whether he or she is observing the signal or a message, but our focus on pure strategy equilibria satisfying the listening perfection refinement implies that any player who assigns any positive probability to the message he or she has being a truthful one assigns it probability one of being truthful. Accordingly, in equilibrium, messages not equal to φ are epistemologically equivalent to the signal itself.

[14] Note that using the listening perfection refinement requires the inclusion of the third "no message" message, φ. In particular, without such a third message being available to each agent, there might be no equilibrium satisfying the

Once one agent does not have an incentive to be truthful with the next, the messaging breaks down for *all* subsequent agents in the network. For any given non-crossing loop network G and any given source $\sigma \in N$, let $M(\sigma; G)$ denote the maximal number of truthful messages that can be sent in a pure strategy equilibrium, given σ and G. We refer to any network G and source σ as supporting a *completely truthful* equilibrium is $M(\sigma; G) = n - 1$. When the context is clear, we omit the conditioning of this statement on σ (*i.e.*, when there is no risk of confusion, we implicitly hold the source fixed for the purposes of discussion).

3 Equilibrium Analysis

We first derive the incentive compatibility conditions for any given agent $i \in N$ to be truthful with any other given agent $j \in N$, supposing that messaging concludes and policymaking occurs immediately after i's message. Under this presumption, the actions of every agent other than j will be invariant to i's message, and presuming that j believes that (*i.e.*, will make policy as if) i will be truthful. the incentive compatibility conditions for i are as follows. For a given network G and source $\sigma \in N$, let $e^*(\sigma; G)$ denote a pure strategy equilibrium of the game following realization of $\sigma \in N$ as the source containing the maximal ($M(\sigma; G)$) number of truthful messages.[15] Furthermore, for any non-crossing loop network G, source $\sigma \in N$, and player $i \in N$, let

$$\psi(i; \sigma, G) \equiv \{j \in N : j \text{ is on all paths from } i \text{ to } \sigma\}$$

denote the set of agents "between i and the source σ" on G.[16] For the source σ, $\psi(\sigma; \sigma, G) = N \setminus \sigma$: the set of agents between σ and himself/herself is everybody else. Finally, for any non-crossing loop network G, source $\sigma \in N$, and player $i \in N$, let

$$\mu(i; \sigma, G) \equiv \{j \in N : j \in \psi(i; \sigma, G) \text{ and } j \text{ is truthful in } e^*(\sigma; G)\}$$

denote the set of agents after i that are truthful in $e^*(\sigma; G)$.

For any source $\sigma \in N$ and non-crossing loop network G, the incentive compatibility conditions

refinement, as it might require that some agent j believes that agent i's message is informative even when it is common knowledge that some earlier agent (*e.g.*, the source σ) was using an uninformative messaging strategy.

[15] There are potentially multiple such equilibria, but they are all payoff-equivalent.

[16] The qualifier "on *all* paths" is necessary because everybody is on multiple paths between any two other agents, because G is a loop.

for truthful messaging by any agent $j \in N$ are:

$$\sum_{i \in \mu(j;\sigma,G)} \alpha_i (\beta_j - \beta_i)^2 \leq \sum_{i \in \mu(j;\sigma,G)} \alpha_i \left(\beta_j - \beta_i - \frac{1}{3}\right)^2, \text{ and}$$

$$\sum_{i \in \mu(j;\sigma,G)} \alpha_i (\beta_j - \beta_i)^2 \leq \sum_{i \in \mu(j;\sigma,G)} \alpha_i \left(\beta_j - \beta_i + \frac{1}{3}\right)^2.$$

These are satisfied if and only if[17]

$$\left| \frac{\sum_{i \in \mu(j;\sigma,G)} \alpha_i (\beta_j - \beta_i)}{\sum_{i \in \mu(j;\sigma,G)} \alpha_i} \right| \leq \frac{1}{6}. \quad (3)$$

This is equivalently rewritten as

$$\left| \beta_j - \frac{\sum_{i \in \mu(j;\sigma,G)} \alpha_i \beta_i}{\sum_{i \in \mu(j;\sigma,G)} \alpha_i} \right| \leq \frac{1}{6}.$$

For any agent $i \in N$, let $\hat{\beta}_i(\alpha) \equiv \frac{\sum_{i \in \mu(j;\sigma,G)} \alpha_i \beta_i}{\sum_{i \in \mu(j;\sigma,G)} \alpha_i}$ denote the weighted mean of preference biases for all agents other than i. In the canonical baseline case in which all agents have equal decision-making weight—$\alpha_i = \alpha_j = 1$ for all $i, j \in N$—$\hat{\beta}_i(\alpha)$ simply reduces to the mean of the preference biases of all agents other than i.

With this in hand, and in order to understand how the network structure—*i.e.*, the ordering of the loop—"matters" in this environment, it is illustrative to first consider when it *doesn't* matter. The next proposition characterizes this formally.

Proposition 1 *Suppose that (N, α, β) are such that $\hat{b}_j \leq \frac{1}{6}$ for all j. Then*

$$M(\sigma; G) = n$$

for all sources σ and for all non-crossing loop networks G.

Proposition 1 identifies situations in which the loop doesn't matter—these are exactly the situations in which, regardless of the source σ, the group could simply (and credibly) sit in a room, allow the source to announce his or her signal, and presume the announcement is truthful. This point is the real import of Proposition 1: the existence of "a loop"—*i.e.*, private sequential messaging between pairs of agents as opposed to simple, one-shot public messaging within the group—is arguably a method of (partially) overcoming preference divergence within the group in pursuit of information aggregation by/between at least some of the members of the group. In the next section we demonstrate more specifically how a loop can do this. In particular, the loop provides a special type of credible commitment to share information broadly.

[17] Inequality (3) understandably mirrors, but does not duplicate, Inequality (4) in Hagenbach and Koessler (2010).

4 Examples

In this section, we present three examples that illustrate the impact of network structure and position on sequential communication.

4.1 The Importance of Mediators

The importance of the loop structure for Proposition 1 can be demonstrated by considering the following simple three player example. Specifically, it compares the loop structure with the canonical "private messaging" protocol in which the first message-sender (the source) can communicate privately and separately with the other agents. The (correctly chosen) loop network structure in this example provides credibility to communication by this agent precisely because it requires that the source communicate to his or her ally

Example 1 (Loop Matters) Suppose that $N = \{1,2,3\}$, with

$$\alpha_1 = 1, \quad \beta_1 = 0,$$
$$\alpha_2 = 1, \quad \beta_2 = 0.1,$$
$$\alpha_3 = 1, \quad \beta_3 = 0.2.$$

Consider the case in which player 1 is the source: $\sigma = 1$, and compare the non-crossing loop network $G = (1,2,3)$ with the non-loop network $G' = \{(1,2),(1,3)\}$. In the non-loop network, player 1 can send a private message to player 2 and a (possibly different) private message to player 3. We denote the message from player 1 to player 2 in G' as $m_{1,2}$ and that from player 1 to player 3 by $m_{1,3}$.

Working backwards, straightforward and iterative application of inequality (3) verifies that a completely truthful equilibrium exists in G. Turning to the private message network, incentive compatibility in G' is relevant only for player 1, and involves the following two inequalities:

$$|\beta_1 - \beta_2| \leq \frac{1}{6}, \text{ and} \tag{4}$$
$$|\beta_1 - \beta_3| \leq \frac{1}{6}. \tag{5}$$

Clearly, inequality (4) is satisfied, but (5) is not. That is, when allowed to message separately to players 2 and 3, player 1 can be credibly truthful only to player 2: player 3 will not believe a cheap-talk message from player 1, and player 1 will accordingly send $m_{1,3} = \varphi$ in equilibrium. Finally, note that $G'' = (1,3,2)$ also possesses a completely truthful equilibrium. This is because of the symmetry of the incentives of players 2 and 3 in terms of their dyadic communication: because they are the final two players in the loop, if one will be truthful with the other, then the converse holds as well. △

One interesting implication of Example 1 can be seen by considering the choice of network design. In this case, *all* agents would strictly prefer the loop structure to the private messaging protocol. Thus, for example, all three players would strictly prefer to adopt some (unmodelled here) institutional protocol that prohibited players 1 and 3 from communicating. Furthermore, this preference is adopted precisely because *both* players 1 and 3 prefer that player 1 communicate truthfully (albeit indirectly) with player 3!

4.2 The Importance of Position

We now provide an example demonstrating the sensitivity of credible communication to the exact locations/ordering of the individuals within a communication network. In addition, the example clearly demonstrates that the credibility of truthfulness between any pair of agents within this framework is a multidimensional phenomenon. While policy is unidimensional—so that preference divergence between any pair of agents i and j can be summarized by a single number (*e.g.*, $|\beta_i - \beta_j|$)—-the relevance of this divergence is proportional to the listening agent's authority (*i.e.*, α_j). Of course, as the number of agents within the network grows, the space required to faithfully capture any given individual's incentives grows as well, as every subsequent individual in the network potentially influences the incentives of earlier message-senders to be truthful.

Example 2 (Order Matters) Suppose that $N = \{1,2,3,4\}$, with

$$\begin{aligned}
\alpha_1 &= 1, \quad \beta_1 = 0.1, \\
\alpha_2 &= 2, \quad \beta_2 = 0, \\
\alpha_3 &= 5, \quad \beta_3 = 0.2, \\
\alpha_4 &= 2, \quad \beta_4 = 0.1.
\end{aligned}$$

Consider the case in which player 1 is the source: $\sigma = 1$, and compare $G = (1,2,3,4)$ with $G' = (1,3,2,4)$. Working backwards, straightforward application of inequality (3) verifies that truthfulness is incentive compatible for player 3 to be truthful in G. In G', the same calculation verifies that truthfulness is incentive compatible for player 2.

Given these steps, consider the incentive to 2 to be truthful in G and the same incentive for player 3 in G', presuming that players 3 (in G) and 2 (in G') are being truthful to player 4. Inequality (3) for player 2 in G is violated, whereas it is not violated for player 3 in G'. Finally, inequality (3) is satisfied for player 1 in G', presuming that all other players are being truthful. Thus, $M(1;G') = 3$: a completely truthful equilibrium exists for this network structure, whereas such an equilibrium does not exist for G.[18]

[18] Note that inequality (3) is also satisfied for player 1 in G under the presumption that player 2 is *not* being truthful and inequality (3) is violated for player 2 in G under the presumption that player 3 is *not* being truthful. Thus, $M(1;G) = 1$.

The distinction between the two loops in this example is the relative power of an extremist, player 3. By placing him or her "early" in the loop, one avoids "tempting" player 2 to lie about his or her information in an attempt to manipulate player 3's individual policy decision. Furthermore, this ameliorating effect of reversing 2 and 3's positions in the loop is possible only because of the presence of a fourth individual later in the loop (in this example, player 4). It is straightforward to verify that neither of the networks $\{1, 4, 3, 2\}$ or $\{1, 4, 2, 3\}$ support a completely truthful equilibrium. △

4.3 The Importance of Inclusion/Exclusion

The next example is a type of converse of Example 1. While that example demonstrated the importance of the sequential structure of the network in supporting credible communication among three agents, the following example demonstrates the potential credibility problems that can be created by the inclusion of one or more individuals within the communication network. That is, as mentioned in the introduction, just as a communication network can mitigate credibility problems by providing an individual i with an incentive to be truthful with another individual j precisely because of individual i's desire for j to possess i's information so that j can then pass along the information to a third individual, k, the presence of a third individual to whom j might communicate after i communicates with j can overturn i's incentive to be truthful with j.

Example 3 (Exclusion Can Help) Suppose that $N = \{1, 2, 3, 4\}$, with

$$\begin{aligned} \alpha_1 &= 1, \quad \beta_1 = 0.00, \\ \alpha_2 &= 1, \quad \beta_2 = 0.08, \\ \alpha_3 &= 1, \quad \beta_3 = 0.16, \\ \alpha_4 &= 1, \quad \beta_4 = 0.30. \end{aligned}$$

In this setting, each individual is equally powerful, and straightforward calculations verify that *none* of the non-crossing loop networks support a completely truthful equilibrium. That is, regardless of the ordering of the loop and regardless of the source, some agent always has an incentive to obfuscate. More importantly, this example demonstrates how exclusion can help. In particular, consider the non-crossing loop network $G = \{1, 2, 3, 4\}$. In this network, players 2 and 3 each have an incentive to be truthful and, accordingly, any equilibrium satisfying the listening perfection refinement involves $m_2^*(\eta_2) = \eta_2$ and $m_3^*(\eta_3) = \eta_3$ (*i.e.*, in such an equilibrium players 3 and 4 will believe players 2 and 3, respectively). However, this truthfulness by players 2 and 3 implies that player 1 has a strict incentive to not truthfully reveal his or her signal (he or she *always* strictly prefers revealing $m_1 = 0$ to $m_1 \in \{1, \varphi\}$ if player 2 believes that 1 is being truthful): the divergence of preferences between players 1 and 4 is too great to support credible cheap-talk transmission from player 1 of information that he or she believes will ultimately be revealed to player 4. Thus, the essentially unique equilibrium satisfying the listening perfection refinement involves player 2 not updating his or her beliefs based on player 1's message, player 1 choosing $m_1(eta_1) = \varphi$ at both information sets $\eta_1 \in \{0,1\}$, and every other player playing a truthful strategy and believing that all players other player 1 are being truthful. Accordingly, $M(1,G) = 0$: in any equilibrium of this situation satisfying the listening perfection refinement discussed in Section 2.1, *no* information is revealed.

To see that exclusion helps, note that the network structure $G_{-4} \equiv \{1,2,3\}$, where player 4 is excluded, does support a completely truthful equilibrium: $M(\sigma, G_{-4}) = 2$ for each $\sigma \in \{1,2,3\}$. Of course, exclusion of agent 4 is an imperfect solution to the information aggregation problem. This is for two reasons. First, player 4's policy choice is uninformed whenever $\sigma \neq 4$. Second, the policy choices made by players 1, 2, and 3 are uninformed whenever $\sigma = 4$. △

Example 3 illustrates yet another area for exploration and extension of the model. In particular, one could model the trade-off between any individual i's *ex ante* "informational quality" (modeled as the probability that he or she is the source, p_i) with the likelihood of the other individuals being chosen as the course, weighted against the incentive/credibility problems that i's presence in the network might cause. Of course, some individuals might be clearly dominant to include in the network because their preferences are "moderate" relative to the other individuals. Less obviously, as indicated by Example 2, the relative value of including an individual i is not solely a function of p_i: even if $p_i = 0$, a moderate individual might be strictly valuable because their presence can engender credible truthfulness within a larger set of the individuals, N. Finally, and even more subtly, Example 2 also implies that the value/cost of including any given individual will depend on the degree to which the order of the loop is fixed (for example, agents can only be "knocked out" of the loop, but not shuffled around) or is itself and institutional design choice (*e.g.*, not only might the institutional designer have complete freedom to include and/or position individuals within the network, he or she might also have the ability to create multiple independent loops, *etc.*). Needless to say, there are many possibilities and modeling choices offered by extensions along these lines.

5 Conclusion

To be written.

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